DISCHARGE OF A POWDER – AIR MIXTURE WITH A HIGH CONCENTRATION OF POWDER FROM A VESSEL UNDER PRESSURE

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The discharge of a mixture through a short spout is analyzed. A formula for determining the velocity of particles has been derived and experimentally checked. The conditions of critical discharge are analyzed and a formula is proposed for determining the critical air velocity in a mixed stream.

Not enough consideration has been given to pneumatic transport of powders in a high-concentration stream. No procedure for designing such pneumatic transport exists at present, which is explained by the novelty of this method on the one hand and by the overall status of the multicomponent flow theory [1].

In order to develop a design procedure, it is necessary to answer several questions concerning the ratio of component velocities and the total friction losses in the case of a complex transport route. Until these questions have been finally resolved, it would be of interest to investigate simplified schemes with a minimum number of unknowns.

The authors consider a device shown in Fig. 1. Compressed air is fed underneath the distributor grid 2 into a hermetically closed vessel 1 full of powder, whereupon the air and powder mixture discharges through a short cylindrical spout 3 into a medium at atmospheric pressure. Under steady-state conditions the pressure p_k in the vessel as well as the air flow rate $G_a^{\prime}$ and the powder flow rate G_p remain constant. It is desired to determine the powder discharge velocity.

In order to use the mathematic apparatus available from the theory of continuous media, one replaces the discrete quantities with continuous ones by the method of space–time averaging [1].

The fundamental equation will be obtained from the Law of Energy Conservation for a steady stream [2]. The energy per second transmitted through any section of the stream is

$$E_m = G_p\left(u_p + \frac{p}{\rho_p} + \frac{\omega_p^2}{2} + gh\right) + G_a\left(u_a + \frac{p}{\rho_a} + \frac{\omega_a^2}{2} + gh\right). \tag{1}$$

Passing to an infinitesimally near section in the stream,

$$dE_m = dQ_e, \tag{2}$$

where Q_e is the external heat per second supplied to the stream through the walls of the given segment.

From the First Law of Thermodynamics follows

$$G_p\,du_p + G_a\,du_a = dQ_e + dQ_f - G_p p d\left(\frac{1}{\rho_p}\right) - G_a p d\left(\frac{1}{\rho_a}\right), \tag{3}$$

where dQ_f is the heat per second dissipated inside the given stream segment and equal to the work of friction.

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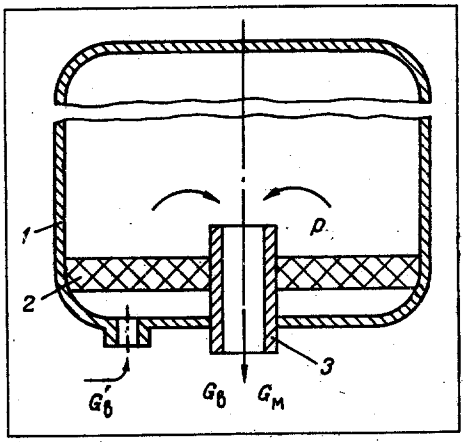

Fig. 1. Schematic diagram of the apparatus.

Inserting dQ_e from (3) and dE_m from (1) into (2), and considering that

$$d\left(\frac{p}{\rho_a}\right)-pd\left(\frac{1}{\rho_a}\right)=\frac{dp}{\rho_a},$$

we obtain

$$G_a\frac{dp}{\rho_a}+G_p\frac{dp}{\rho_p}+G_p d\left(\frac{\omega_p^2}{2}\right)+G_a d\left(\frac{\omega_a^2}{2}\right)$$
$$+(G_p+G_a)gdh+dQ_f=0. \tag{4}$$

Assuming, with little error, the expansion of air to be isothermal when the powder concentration is high [3], we then integrate Eq. (4) from the stream section inside the vessel, where $\omega_a = \omega_p = 0$, $h = 0$, $p = p_k$ to an arbitrary section:

$$\beta RT\ln\frac{p_k}{p}+\frac{1-\beta}{\rho_p}(p_k-p)=\beta\frac{\omega_a^2}{2}+(1-\beta)\frac{\omega_p^2}{2}+gh+L,\ldots \tag{5}$$

Here L is the energy loss per kilogram of mixture.

We note that the left-hand side of Eq. (5) physically signifies the useful external work per kilogram of mixture, which is determined only by the pressure drop and the quantity β. Into the right-hand side of Eq. (5) may, generally, be entered any energy losses, also those occurring when the velocities and the concentrations of the components are distributed arbitrarily over a stream section.

In our problem expression (5) can be simplified by the omission of the small quantities gh, $\beta(\omega_a^2/2)$, and L. Thus, for $p_k > 1.5$ atm abs., $p = 1$ atm abs., and $\rho_p \approx 3000$ kg/m³ the left-hand side amounts to more than 120 m²/sec² with gh less than 5 m²/sec² (when the fluidization zone in the vessel is not higher than 0.5 m above the spout exit section). With $\omega_a \approx 2\omega_p$ and $\beta < 0.01$, according to test data, we find that $\beta\omega_a^2/2$ amounts to not more than 4% of $(1-\beta)(\omega_p^2/2)$. The friction losses have been neglected here, on the basis of the analogy to a discharge of homogeneous fluids through spouts.

With p_1 denoting the spout exit pressure, which can be higher than the ambient pressure, we have then

$$\omega_p^2\approx 2\left(\frac{\beta}{1-\beta}RT\ln\frac{p_k}{p_1}+\frac{p_k-p_1}{\rho_p}\right). \tag{6}$$

This expression is, except for the factor $(1-\beta)$, identical to the formula in [4] for gas–liquid mixtures with equal velocities of the components.

Let us compare formula (6) with other known expressions for the discharge velocity of a pseudofluid from a vessel under pressure.

Urban in [5] shows the relation

$$\omega_p^2=\frac{p_k-p_1}{\rho_p(1-\varepsilon_1)},$$

where ε_1 is the porosity at the exit section of the spout.

This formula can be derived by integrating (4) with the three last terms on the left-hand side omitted and with $G_a = \varepsilon_1\rho_a F\omega_a$ and $G_p = (1-\varepsilon_1)\rho_p\omega_p F$, assuming here that $\omega_a = \omega_p$ and F = const. With this assumption, expression (4) will yield Urban's formula instead of formula (6).

We note that, according to the results shown here, subsequently, the condition of equal velocities does not apply. The probability and the possibility of satisfying the second condition (of a constant stream cross-section area) are small, because the flowing mixture forms a funnel inside the vessel, especially at high pressures, and the test data indicate that the velocity of particles is quite high already at the spout entrance.

On the basis of a test data evaluation, Massimilla [6] has derived a formula for the flow rate of fluidized powder discharging from a vessel. It has been shown in [7] that the discharge velocity in Massimilla's formula is determined by the relation

$$\omega_p^2 = 2\frac{p_k - p_1}{(1-\varepsilon_k)\rho_p},$$

where ε_k is the porosity of the pseudofluid inside the vessel.

The last expression can be obtained by integrating the equation of energy conservation for some constant-density fluid:

$$d\left(\frac{\omega_p^2}{2}\right) = -\frac{dp}{(1-\varepsilon_k)\rho_p},$$

with $(1-\varepsilon_k)\rho_p$ considered constant. Since, during discharge, the density of a mixture must vary with the pressure, hence the assumption of a constant density renders Massimilla's formula applicable to low pressures only.

In comparison with the formulas given by Massimilla and Urban, we note that formula (6), not being limited with regard to pressure nor requiring a constant channel section and equal velocities of the components, is more general. Since friction is disregarded in formula (6) while the pressure on the mixture components is assumed equal and independent of the distance between particles, this expression is applicable only to a complete fluidized powder around the exit opening.

We note that, in order to calculate the discharge effectiveness, one must know not only ω_p but also ε_1, the latter being a function of the component velocities ratio. Inasmuch as the laws governing this ratio in a high-concentration stream have not yet been sufficiently well explored, such calculations are difficult.

The validity of formula (6) was checked experimentally on the apparatus shown in Fig. 1.

The hermetically closed vessel 1 contained approximately 70 kg of powder concentrate with an average grain size 58 μ and a density of 3200 kg/m^3. The cylindrical spout 3 had the minimum necessary (structurally) length of 85 mm. The diameter of the exit channel was made small, 9 mm only, in order to sufficiently extend the steady-state discharge time. The upper spout end was positioned at 30 mm from the plane of the gas distributor mesh, the latter made of technical-grade felt. Oscillographic measurements included pressure p_k in the vessel, pressure p_1 at the spout exit section, air flow rate G_a', change in the vessel weight, and jet momentum at the exit section. The last item was measured with a strain–gage strip as in [8].

For measuring p_1, at 1.5 mm from the exit section, holes were provided in the channel connecting it to the test chamber. The ambient pressure was at atmospheric level, the powder temperature was $T \approx 290$°K.

The test results are shown in Table 1 (columns 2-6).

The theoretical values of velocity ω_{pT} of particles at the exit section have been calculated by formula (6) from the test values of p_k, p_1, and β, and they are listed in column 13. A comparison between ω_p and ω_{pT} values shows a satisfactory agreement at pressures $p_k > 2.5 \cdot 10^5$ N/m^2. The wide divergence at lower pressures (tests No. 1 and No. 2) can, apparently, be explained by the more significant stray energy losses at the entrance, for example.

We must emphasize an important, in our opinion, fact: in the tests with $p_k > 2.5 \cdot 10^5$ N/m^2, $p_1 > p_0$. This kind of inequality in a stream of "pure" air would apply the critical discharge velocity, which is equal to the velocity of sound. Since in our tests the air velocity was much lower than the velocity of sound, it would be interesting to estimate the critical air velocity in the mixture.

Such a velocity, assuming that the stream fills the entire channel section, can be determined on the basis of the following considerations.

In passage along the channel into an infinitesimally near section, the volume per second of mixture increase by dV_m due to the increment of air volume. In order to pass this additional volume through the section, the air velocity and other related to it parameters must also change. The change in air velocity depends on the part of the pressure head dp used for acceleration. The maximum possible increment of volume per second which is due to the increment of velocity will occur when there are no losses and the entire pressure head dp is used for acceleration. This maximum possible increment of volume per second

TABLE 1. Test Results and Calculated Values

Item number	$p_k \cdot 10^{-5}$ N/m²	$p_1 \cdot 10^{-5}$, N/m²	$G'_a \cdot 10^4$, kg/sec	G_p, kg/sec	S, N	$G_a \cdot 10^4$, kg/sec	$\beta \cdot 10^4$	ω_p, m/sec	ε_1	ω_a, m/sec	ω_a/ω_p	ω_{pT}, m/sec
1	2	3	4	5	6	7	8	9	10	11	12	13
1	1,54	0,981	14,2	0,53	4,9	11,8	22,3	9,3	0,778	19,5	2,1	11,3
2	2,14	0,981	21,7	0,67	9,32	17,2	25,6	13,9	0,829	26,8	1,92	19,6
3	2,58	1,03	34,6	0,72	17,8	30,8	41,7	24,4	0,867	43,5	1,78	27
4	2,86	1,08	41,5	0,825	24,2	38,4	50,3	28,7	0,865	52	1,81	30,3
5	3,3	1,16	46,6	0,89	24,7	37,4	41,8	26,6	0,846	48,4	1,82	28,9
6	4,03	1,19	70	1,0	36,8	56	56	35,4	0,862	69,5	1 96	36
7	4,23	1,17	70	0,98	37	57	58,2	36,6	0,87	72,6	1,98	37,7
8	4,33	1,21	72	1,0	40,7	58	58	39,3	0,868	70,5	1,80	37,8
9	4,6	1,23	76	1,04	44	60	57,5	40,8	0,864	72	1,76	38,2
10	4,83	1,24	79	1,06	43,7	63	59,5	39,7	0,865	73,6	1,85	39,2
11	5,31	1,32	92	1,12	48,2	72	64	41,1	0,866	81,5	1,98	41,6

Note. $G_a = G'_a - \frac{G_p p_k}{\rho_p RT}$; $\omega_p = \frac{S - (p_1 - p_0) F}{G_p}$; $\varepsilon_1 = 1 - \frac{G_p}{\omega_p \rho_p F}$; $\omega_a = \frac{G_a RT}{\varepsilon_1 p_1 F}$

will be denoted by dV'_m. Obviously, flow can occur only when $dV'_m > dV_m$, because part of dp is lost on friction, lift, etc. The limiting case $dV'_m = dV_m$ corresponds to the critical mode. If such a flow mode were to prevail at some intermediate section of the cylindrical channel, therefore, then a farther flow would be impossible. In order to determine the critical velocity of mixture components, therefore, it is necessary to find dV'_m and dV_m and to equate them.

The volume per second of mixture is

$$V_m = \frac{G_p}{\rho_p} + \frac{G_a}{\rho_a}, \tag{7}$$

where

$$G_a = \rho_a \omega_a \varepsilon F \tag{8}$$

and

$$G_p = \rho_p \omega_p (1 - \varepsilon) F. \tag{9}$$

From (7) we find

$$dV_m = G_a d\left(\frac{1}{\rho_a}\right)$$

or, considering (8),

$$dV_m = -\omega_a \varepsilon F \frac{d\rho_a}{\rho_a}. \tag{10}$$

In order to determine dV'_m, we write (7) with (8) and (9) as p

$$V_m = \omega_a \varepsilon F + \omega_м (1 - \varepsilon) F,$$

and from here, assuming $\omega_p F(1-\varepsilon) = \text{const}$, we find

$$dV_m = \varepsilon F d\omega_a + \omega_a d(\varepsilon F).$$

Into this expression we insert $d\omega_a$ found from the equation of momentum change for the mixture, without losses,

$$G_a d\omega_a + G_p d\omega_p = -F dP.$$

After this substitution and transformations with (8) and (9) taken into account, we obtain

$$dV'_m = \omega_a d(\varepsilon F) + \frac{1}{\omega_a \rho_a}\left[-F dp - \omega_p \rho_p F (1 - \varepsilon) d\omega_p\right]. \tag{11}$$

Inserting into (11)

$$F(1 - \varepsilon) d\omega_p = -\omega_p d\left[(1 - \varepsilon) F\right],$$

which follows from (9), and introducing $\omega_p = \omega_a/k$, where k is a variable quantity (function of the flow parameters), we then equate (10) and (11). Appropriate transformations will yield the critical, i.e., the maximum possible velocity $\omega_a = \omega_{cr}$:

$$\omega^2_{a,cr} = \frac{dp}{d(\varepsilon\rho_a) + \rho_a\varepsilon\frac{dF}{F} + \frac{\rho_p}{k^2}\left[(1-\varepsilon)\frac{dF}{F} + d(1-\varepsilon)\right]}. \tag{12}$$

For the case where $\varepsilon = 1$ and F = const (flow of "pure" air through a cylindrical channel), expression (12) becomes the well-known formula

$$\omega^2_{a,cr} = \frac{dp}{d\rho_a} = a^2_a,$$

where a_a denotes the velocity of sound in air.

Separating (8) and (9), then letting $u = G_p/G_a$, we obtain

$$\varepsilon = \frac{\rho_p}{\rho_p + k\mu\rho_a}. \tag{13}$$

We now insert (13) into (12), let dF = 0 (our test conditions), then divide the numerator and the denominator of (12) by dp. After differentiation with $dp/d\rho_a = a_a^2$ and appropriate transformations, we obtain

$$\omega^2_{a,cr} = \frac{a^2_a\left(1 + \frac{k\mu\rho_a}{\rho_p}\right)^2}{1 + \frac{\mu}{k} + \mu\rho_a\left(\frac{1}{k} - \frac{\rho_a}{\rho_p}\right)\frac{dk}{dp}}. \tag{14}$$

Letting k = const, we have

$$\omega^2_{a,cr} = \frac{a^2_a k}{k+\mu}\left(1 + \frac{k\mu\rho_a}{\rho_p}\right)^2.$$

The last expression differs from the approximate formula obtained in [3] for the critical velocity by the factor in parenthesis.

When k = 1, expression (14) yields

$$\omega^2_{a,cr} = a^2_a\frac{\left(1 + \frac{\mu\rho_a}{\rho_p}\right)^2}{1+\mu} = a^2_a\frac{\beta}{\varepsilon^2}. \tag{15}$$

Expression (15) for the critical air velocity in a mixture is identical to the formula for the velocity of sound in a two-component mixture with equal velocities of the components [9, 10].

We note that calculations in [9] show the velocity of sound in the mixture to be very low when k = 1. Thus, $\omega_{a,cr} = 12.1$ m/sec at $\varepsilon = 0.7$, p = 1 atm abs., and $\rho_p = 3200$ kg/m^3 (apatite concentrate).

Assuming that the air velocity was critical in our tests, where $p_1 > p_0$, and inserting into (14) the value of air velocity as well as the values of other quantities obtained in these tests, we find that dk/dp is negative and, consequently, the ratio ω_a/ω_p increases toward the exit section.

NOTATION

p_k is the pressure inside the vessel;
p_1 is the pressure at the spout exit;
p_0 is the ambient pressure;
p is the pressure at any stream section;
G'_a is the rate (per second) of air flow into the vessel;
G_a is the rate (per second) of air flow in the mixture;
G_p is the rate (per second) of particle flow;
E_m is the energy of mixture;
u_p is the internal energy (per kilogram) of particles;

u_a is the internal energy (per kilogram) of air;
ρ_p is the density of powder material;
ρ_a is the density of air;
ω_p is the true velocity of particles;
ω_a is the true velocity of air;
g is the gravitational acceleration;
h is the height;
Q_e is the external heat;
Q_f is the heat of friction;
L is the energy loss (per kilogram) in mixture;
R is the gas constant;
T is the absolute temperature;
S is the jet momentum;
F is the area of channel cross section;
ε is the porosity (ratio of air volume to total volume of mixture);
V_m is the per second volume of mixture;
$\omega_{a,cr}$ is the critical velocity of air in mixture;
a_a is the velocity of sound in mixture;
$\mu = G_p/G_a$.

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